

## Upper And Lower Bounds

(Usually I would say that you should round answers to 3 sig figs etc, but just this once I would like you to write answers showing all digits on your calculator display. Thanks.)

1. Write down the lower bound and upper bounds for the following:

- (a) 110 (correct to the nearest integer). 109.5, 110.5
- (b) 110 (correct to the nearest ten). 105, 115
- (c) 110 (correct to the nearest five). 107.5, 112.5
- (d) 123.8 (correct to 1dp). 123.75, 123.85
- (e) 4500 (correct to 3 sf). 4495, 4505

2. If  $x = 3.6$  and  $y = 4.1$ , both correct to 1 decimal place, find the upper bound and lower bounds for

- (a)  $x + y$ . 7.6, 7.8
- (b)  $xy$ . 14.3775, 15.1475
- (c)  $\frac{1}{x} + \frac{1}{y}$ . 0.514936458161, 0.528603721092
- (d)  $\frac{1}{x} - \frac{1}{y}$ . 0.027059022493, 0.040726285423

3. A sprinter is running the 100 metres dash. The length of the track is 100 m (correct to the nearest metre). Her time to finish the race is 12.1 seconds (correct to 1 dp). Find the upper bound for her average speed. 8.320248962656

4. A bowl is made out a solid hemisphere of radius 6 cm (correct to nearest cm) with another hemisphere of radius 5.1 cm (correct to 1 dp) carved out. If the density of the material is  $2 \text{ g/cm}^3$ , find the upper bound for the mass of the bowl in grams. 610.8821678918

5. A rectangular swimming pool is 25 metres by 10 metres (both correct to the nearest metre). A path is constructed around the edge of the pool of width 1 metre (correct to nearest metre). [i.e. rectangle around a rectangle.]. Find the lower bound for the area of the path. □

6. A solid cone has radius 12.5 cm (correct to 3 sf) and perpendicular height 20 cm (correct to nearest cm). Find the lower bound for its total surface area. □

7. A solid metal cylinder has height 2 metres and radius 1 metre (both correct to the nearest metre). It is melted down and cast into 10 spheres. Find the upper bound for the radius of each individual sphere.  $r_{\max} = \frac{3}{4}$

8. If  $y = 2^x$  and  $x = 2.7$  (correct to 1dp), find the upper bound for  $y$ . 6.72717132203

9. If  $y = \frac{1}{x+1}$  and  $x = 7$  (correct to nearest integer), find the upper bound for  $y$ . □

10. If  $y = \frac{1}{x+1}$  and  $y = 7$  (correct to nearest integer), find the upper bound for  $x$ . □

11. If  $y = 3^{-x}$  and  $x = 0.8$  (correct to 1dp), find the lower bound for  $y$ . 2.544210651641

12. If  $y = 0.6^x$  and  $x = 1.1$  (correct to 1dp), find the upper bound for  $y$ . 0.584869282782

13. If  $y = x^2 - 2x + 1$  and  $x = 1$  (correct to the nearest integer), find the lower bound for  $y$ .

0 (exactly)

14. If  $y = 2 \sin x$  and  $x = 39$  (correct to nearest integer), find the upper bound for  $y$ .

1.272156440556

15. If  $y = 3 \cos x$  and  $x = 39$  (correct to nearest integer), find the upper bound for  $y$ .

2.347824470557

16. If  $y = \tan x$  and  $x = 100$  (correct to nearest 25 degrees), find the upper bound for  $y$ .

Unbounded (plus infinity in dreadful terms).